SOLUTIONS

Glasses

1a) The volume of Glass 1 = $\pi \times 3 \times 3 \times 6 = 54\pi = 170$ cm^3.

1b) The volume of the hemisphere = $(4\pi \times 3^3) \div 6 = 18\pi = 56.5$.

The volume of the cylinder = $\pi \times 3^2 \times 3 = 27\pi = 84.8$.

Total volume of Glass 2 = $45\pi = 141$ cm^3.

1c) Using the Pythagorean Theorem, the height of the cone is $\sqrt{7^2 - 3^2} = \sqrt{40} = 2\sqrt{10}$.

The volume of Glass 3 = $(\pi \text{ x } 3^2 \text{ x } 2\sqrt{10}) \div 3 = 6\sqrt{10}\pi = 60$ cm^3.

2. The volume of liquid in the half-full Glass 2 is $141 \div 2 = 70.5$.

The volume of liquid in the cylinder = $70.5 - 56.5 = 14$.

$14 = \pi \times 3^2 \times$ height in cylinder.

Height in cylinder = $14 \div 9\pi = 0.5$.

The total height = 3 + 0.5 = 3.5 cm.

3. Glass 4 is composed from a cylinder and cone. While we do not yet have enough information to deduce the formula, it is possible to rule out three of the formulas on grounds that the dimensions of the formulas are incorrect:

$\frac{1}{6}\pi dh$	Only two lengths are multiplied so this has the dimension of an area.
$\frac{1}{6}\pi d^2h^2$	Four lengths are multiplied, so this is not a volume either.

Both $\frac{1}{6}\pi d^2h$ and $\frac{1}{6}\pi dh^2$ involve multiplying together three lengths, but $\frac{1}{6}\pi dh^2$ involves the square of the height and so, cannot be correct.

The correct formula is therefore $\frac{1}{6}\pi d^2h$.

Glasses: Extension problems

1. This question is intended to encourage the discussion of dimensional analysis.

When lengths are combined by addition we obtain another length.

If two lengths are multiplied we obtain an area.

If three are multiplied we obtain a volume.

2. The volume of the Glass = volume of cylinder + volume of cone

$$= \pi\left(\frac{d}{2}\right)^2\left(\frac{h}{2}\right) + \frac{1}{3}\pi\left(\frac{d}{2}\right)^2\left(\frac{h}{2}\right)$$

$$= \frac{4}{3}\pi\left(\frac{d}{2}\right)^2\left(\frac{h}{2}\right)$$

$= \frac{1}{6}\pi d^2 h$.

3. When Glass 3 is half full, it will hold 30 cm^3 (from Q1c).

If the height of liquid is h and the radius of the top of the liquid is r then $\frac{1}{3}\pi r^2 h = 30$.

So $\pi r^2 h = 90$. (1)

By similar triangles:

The ratio *height of bowl : radius of bowl* = $h : r = 2\sqrt{10} : 3$

This means that $r = 0.47h$ (2)

Substituting (2) in (1):

$0.225\ h^3 = 29$, and so $h = 5.1$ cm.

The height of liquid will be 5.1 cm.